

Quantum Mechanics, Gravity, and the Multiverse

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Abstract

The discovery of accelerating expansion of the universe has led us to take the dramatic view that our universe may be one of the many universes in which low energy physical laws take different forms: the multiverse. I explain why/how this view is supported both observationally and theoretically, especially by string theory and eternal inflation. I then describe how quantum mechanics plays a crucial role in understanding the multiverse, even at the largest distance scales. The resulting picture leads to a revolutionary change of our view of spacetime and gravity, and completely unifies the paradigm of the eternally inflating multiverse with the many worlds interpretation of quantum mechanics. The picture also provides a solution to a long-standing problem in eternal inflation, called the measure problem, which I briefly describe.

1 Introduction—Why the Multiverse

Why does our universe have the structure we see today? For example, why do the quarks and leptons have the observed masses, and why are there four elementary (electromagnetic, weak, strong, and gravitational) forces acting on them? At some point in the history of elementary particle physics, we hoped that all these questions would be answered once we had figured out the “fundamental theory of nature.” Namely, mathematical consistency of the ultimate theory would not allow any other world than the one we see today. In the past few decades, however, we have gradually been asked—or forced—to consider that this may not be the case: many (if not all) of the structure we observe are due to our very own existence in the huge *multiverse*, a collection of many different universes in which everyday physical laws take different forms.

A shocking revelation which has hugely impacted our thinking came in 1998 when it was discovered that the expansion of the universe is accelerating [1]. Because the gravitational force between two bodies is attractive, the expansion of the universe must be decelerating if it contains only matter (in any form, even dark matter). In order to explain the peculiar phenomenon of accelerating expansion, the universe must be filled with energy with “negative pressure” (called dark energy). The simplest possibility for such a strange entity is energy of the vacuum: the observed acceleration is accounted for if the vacuum has energy density

$$\rho_{\Lambda} \sim 7 \times 10^{-30} \text{ g/cm}^3, \quad (1)$$

which is comparable to the average energy density of matter in the universe, $\rho_{\text{matter}} \sim 3 \times 10^{-30} \text{ g/cm}^3$. A question is why these two totally different entities (matter and vacuum!) are so close in density *in the current universe*. This is very mysterious, especially given that even time dependence of the two quantities ρ_{matter} and ρ_{Λ} are different: $\rho_{\text{matter}} \sim 1/t^2$ and $\rho_{\Lambda} \sim \text{const.}$

In fact, a theoretical estimate of the energy density of the vacuum has been a notoriously difficult problem. Quantum mechanical “corrections” to the vacuum energy are huge—at least about 60 orders of magnitude larger than the size allowed by observation. This problem has been known as the cosmological constant problem, and despite many attempts, it has abhorred a simple theoretical solution [2]. Until recently, many theorists had still been hoping that an yet unknown mechanism will set the vacuum energy to be zero, $\rho_{\Lambda} = 0$, but the discovery of nonzero value in Eq. (1) destroyed this hope. How can the theory know when we—the human species—evolve to the point making cosmological observations, and set the vacuum energy density close to the matter energy density *at that particular moment* in the history of the universe?

Already back in 1980’s, Steven Weinberg realized the difficulty of solving the problem, and considered the possibility that the origin of the smallness of the vacuum energy might be “environmental”: we simply cannot exist if the vacuum energy were (much) larger than the matter energy density *at the time when relevant structures of the universe, such as large galaxies, form*

(which is only within a few orders of magnitude of the timescale of the evolution of the human species) [3]. Suppose there are many “universes,” or large enough spacetime regions, in which the vacuum energy, ρ_Λ , takes different values. Then simple calculations can show that unless $|\rho_\Lambda|$ is within a few orders of magnitude of ρ_{matter} in the current universe (the universe the human species observes), no galaxies, and thus presumably no intellectual observers, form. A prediction of this framework is that, unlike many other attempts trying to achieve $\rho_\Lambda = 0$, *we expect to see nonzero ρ_Λ* since values of $|\rho_\Lambda|$ much smaller than needed for the existence of life are unnatural. In fact, this is what happened in 1998: we discovered accelerating expansion of the universe which can be caused by the vacuum energy density, Eq. (1), that is not much different from the matter energy density in magnitude when intellectual life observed the cosmos.

The assumption of multiple universes may seem too big to swallow based on a single observation of accelerating expansion of the universe (although this is completely consistent with what we have learned throughout our history: our Earth turned out to be only one of several planets in the solar system, which is only one of many such systems in the galaxy, which is again one of many in our local cluster, etc). If we look at the structure of the theory of elementary particle physics and cosmology, however, there are also many “miracles” that seem to be too good for our own existence; for example, only a slight change of certain parameters of the theory seems to lead to a completely sterile world, e.g., that without any interesting chemistry [4]. With the multiverse, these apparent “miracles” have a simple explanation—there are many universes within the multiverse in which physical properties including the value of ρ_Λ are different; and only in those universes in which the conditions are friendly enough for life, an intellectual observer would evolve. Therefore, there is no surprise if the observer finds the structure of physical laws to be “tuned” too good for him/her; otherwise, he/she is simply not there.

Interestingly, the existence of the multiverse has been suggested by theories of elementary particle physics and cosmology. String theory—widely considered to be the best candidate for the ultimate theory of nature—predicts the existence of six extra spatial dimensions beyond the three we experience in our everyday life [5]. In the old days, people viewed this as a nuisance. They simply “hid” these dimensions by postulating that they are too small to see, analogous to the direction on a surface of a thin wire perpendicular to the direction of the extension. These extra dimensions, however, turned out to be a blessing, rather than a nuisance—because the six small dimensions can have a variety of meta-stable configurations, string theory can lead to a variety of four (3 spatial + 1 time) dimensional theories at our length scales, whose properties—including ρ_Λ —depend on the shape and size of the compactified six dimensional space [6]. This plethora of possible different worlds is called the string landscape, and the number of such possible worlds is indeed huge: people’s estimates vary but typically give numbers like $O(10^{1000})$. Moreover, once one meta-stable configuration with $\rho_\Lambda > 0$ is realized, then exponential expansion of space, called inflation, occurs. And it has been known from the 1980’s that inflation is generically future-

eternal [7]: once it occurs, space keeps expanding forever. Again, some people viewed this as an undesired feature, but in the context of the string landscape, it implies that all different four dimensional worlds are indeed physically realized in spacetime, producing the multiverse. The way it works is the following: because of infinite space available, all kinds of “bubbles” having different properties inside are formed in eternally inflating spacetime [8], and each of these bubbles corresponds to a universe with definite physical laws. It is quite suggestive that phenomena many (though not all) people found unwanted, but nevertheless indicated by theory, are exactly the elements needed to realize the multiverse, and hence to solve the cosmological constant problem.

Despite all the good features described above, however, understanding the multiverse in eternally inflating spacetime has been notoriously difficult because of the infinity introduced by the eternal nature of inflation. In this article, I explain this problem—often called the measure problem in eternal inflation [9]—and report recent progress on this issue: quantum mechanics is crucial in understanding the multiverse correctly *even at the largest distance scales* [10, 11]. This leads to a dramatic change of our view of spacetime and gravity, consistently with what we learned about quantum gravity in the past two decades: the holographic principle [12] and black hole complementarity [13]. We will find that this new framework completely unifies the eternally inflating multiverse and the many worlds interpretation of quantum mechanics: these are absolutely the same concept [10]. We will also see that the notion of spacetime is “reference frame dependent” [11], precisely analogous to that of simultaneity in special relativity.

2 Predictivity Crisis in Eternal Inflation

The heart of the problem in eternal inflation is well summarized in the following sentence by Alan Guth [9]: “In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times.” Suppose we want to calculate the relative probability for events A and B to happen. Following the standard notion of probability, we might define it as the ratio of the numbers of times events A and B to happen throughout the whole spacetime

$$P = \frac{N_A}{N_B}. \quad (2)$$

The eternal nature of inflation, however, makes both A and B occur infinitely many times: $N_A, N_B = \infty$. The expression in Eq. (2), therefore, is ill-defined. It seems that we need to “regularize” spacetime to make both $N_{A,B}$ finite, at least at a middle stage of the calculation.

An obvious way to do this is to consider an equal-time cutoff, $t = t_c$, and count only events that occur before this cutoff. Suppose we focus only on some finite spatial region at the beginning. Then, since the numbers of events become infinity only due to those that happen in the infinite future, the cutoff makes $N_{A,B}$, and hence P , finite; see Fig. 1 for a schematic depiction. We can then imagine removing this cutoff by sending $t_c \rightarrow \infty$, and obtain a well-defined answer for P . A

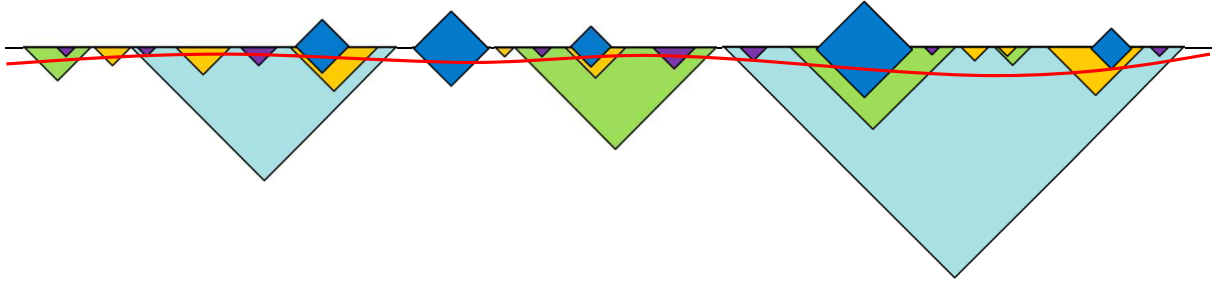


Figure 1: A schematic depiction of the eternally inflating multiverse. The horizontal and vertical directions correspond to spatial and time directions, respectively, rescaled such that the propagation of light is always in a 45° direction. Various regions with the inverted triangle or argyle shape represent different universes, which form in other, parent regions through bubble nucleation processes. While regions closer to the upper edge of the diagram look smaller, it is an artifact of the rescaling made to fit the infinitely large spacetime into a finite drawing—the fractal structure near the upper edge actually corresponds to an infinite number of large (in fact, infinitely large) universes. A fictitious time cutoff, $t = t_c$, is depicted by a red, curved line. The number of universes below this line is finite if we focus on an initially finite spatial region.

problem of this procedure is that the definition of “equal time” is arbitrary. Already in special relativity the concept of equal time depends on an observer, but the situation is much worse in general relativity—there is no way of uniquely introducing the concept of equal time (even an observer dependent one!) for points separated beyond the horizon, especially if the system does not possess any obvious symmetry, which is the case in the eternally inflating multiverse. Indeed, one can show that by carefully devising the cutoff “hypersurface” (a surface of equal time in spacetime), we can obtain any value of P we want—the probability is determined by how we regularize spacetime!

This extreme sensitivity of predictions on the regularization procedure is called the measure problem in eternal inflation. In fact, the problem is much more robust than one might naively think. Suppose there is a meta-stable universe with $\rho_\Lambda > 0$ (more precisely, a meta-stable minimum in the space of quantum fields that has positive potential energy). The conventional wisdom says that if ρ_Λ is smaller than the Planck energy density, $\rho_{\text{Pl}} \simeq 5.1 \times 10^{93} \text{ g/cm}^3$, then the result of general relativity is applicable, so that if the decay rate of such a meta-stable state is small enough, it leads to eternal inflation. This is enough to encounter the problem of predictivity described above—it has nothing to do with the string landscape, the beginning of the universe, or anything like that; in particular, the problem occurs already in a regime where quantum gravitational effects have been *believed to be* unimportant, $\rho_\Lambda \ll \rho_{\text{Pl}}$. This, of course, does not mean that such a belief, i.e. that the solution to the problem does not involve quantum gravity, is correct. In fact, we will see that the solution to the measure problem is intrinsically quantum gravitational.

Another important aspect of the measure problem is that the simplest attempt based on semi-

classical intuition completely fails. Imagine that at some early time the entire universe was in an inflating phase, and that fictitious clocks located at various places are synchronized according to natural time. (Because the inflationary phase has a large symmetry, such natural time—the flat slicing in technical terminology—can be defined.) Now, we can define a cutoff hypersurface as the one on which all the fictitious clocks show the same time, and calculate the probability [14]. In particular, we can calculate the relative probability of us observing a universe with 3K cosmic microwave background (CMB) to that with 2.725K CMB, which gives

$$\frac{N_{T_{\text{CMB}}=3\text{K}}}{N_{T_{\text{CMB}}=2.725\text{K}}} \sim 10^{10^{59}}. \quad (3)$$

Namely, the probability of us seeing a 3K universe is much, much higher than that of seeing a 2.725K universe as we do! This ridiculous conclusion, called the youngness paradox, arises because space expands exponentially in an inflating phase, proportional to $\exp(3Ht)$ with H^{-1} being a microscopic timescale, and the rate of creating universes like our own in such space is constant *per unit physical volume per unit time*. Therefore, the number of universes created at later times increases like crazy, hence giving huge bias towards younger universes when counted at a fixed time defined through the fictitious clocks as described above.

While many proposals have been put forward to solve this and other problems, especially by modifying the way to define the cutoff, they all look rather ad hoc [9]. Indeed, it is extremely uncomfortable that we need to specify the exact way of regulating spacetime *to define the theory*, beyond the basic principles of quantum mechanics and relativity. It seems that something crucial is missing in a way the problem is considered.

3 The Quantum Multiverse

We now argue that the missing ingredient is quantum mechanics. At first sight, this statement sounds trivial—since the process of vacuum decay (a process creating a universe in another universe through a bubble nucleation; see Fig. 1) is probabilistic in the usual quantum mechanical sense, the entire system must ultimately be treated using quantum mechanics. A surprising thing is that it affects our thinking of what spacetime actually is—and hence what the multiverse is—at *distance scales much larger than the Planck length* $l_P \simeq 1.6 \times 10^{-35}$ m, conventionally thought to be the scale only below which quantum gravitational effects become important.

The basic principle we adopt is that *the laws of quantum mechanics are not violated when an appropriate description of physics is adopted*—from the shortest to the largest scales we ever consider. Given the extreme successes of quantum mechanics over the last century, this seems to be a reasonable, and in a sense conservative, hypothesis to take. Then the situation of the eternally inflating multiverse does not seem much different from those in any usual experiments. Suppose

we scatter an electron with a positron, which leads probabilistically to different final states: e^+e^- , $\mu^+\mu^-$, $e^+e^-e^+e^-$, \dots . One might view this as the initial state $|e^+e^- \rangle$ evolving probabilistically into different final states, but this is not true. Since the Schrödinger equation is deterministic, the initial $|e^+e^- \rangle$ state simply evolves deterministically into some final state $\Psi(t = +\infty)$ which, after being decomposed into eigenstates of particle numbers, contains many components:

$$\Psi(t = -\infty) = |e^+e^- \rangle \rightarrow \Psi(t = +\infty) = c_e |e^+e^- \rangle + c_\mu |\mu^+\mu^- \rangle + \dots, \quad (4)$$

where c_e, c_μ, \dots are coefficients calculable according to the Schrödinger equation. The situation for the multiverse must be similar. Starting from a state corresponding to eternally inflating space $|\Sigma \rangle$ at $t = t_0$, it evolves deterministically into some state $\Psi(t)$ at time t which, after being decomposed into states having well-defined semi-classical spatial geometries, contains many components:

$$\Psi(t = t_0) = |\Sigma \rangle \rightarrow \Psi(t) = \sum_i c_i(t) |(\text{cosmic}) \text{ configuration } i \rangle, \quad (5)$$

where the absolute value squared of coefficient $c_i(t)$ should give the probability of finding the universe in cosmic configuration i at time t .

Formulating the multiverse in the form of Eq. (5), however, does not solve any of the problem by itself. What is actually the “multiverse state” $\Psi(t)$? To define a quantum state we need to specify an equal-time hypersurface on which the state is defined, and there is an intrinsic ambiguity in doing this for spatial points separated beyond the horizon. Moreover, even if we follow a region whose spatial extent was initially finite, such a region will grow into an infinitely large spatial region in which an infinite number of observers will arise, so the problem of infinity does persist. We will find below that when the system is treated correctly, the final picture turns out, in fact, like that given in Eq. (5) [10, 11]. To see this, however, we need to understand better quantum mechanics in a system with gravity, which requires a dramatic revision of our view of spacetime.

3.1 Quantum mechanics in a system with gravity

Black holes provide important “laboratories” to test strong gravitational physics. In 1976, Stephen Hawking found a strange phenomenon while studying evolution of evaporating black holes [15]. Suppose we drop some book A into a black hole and observe subsequent evolution of the system from a distance. The book will be absorbed into (the horizon of) the black hole, which will then eventually evaporate, leaving Hawking radiation. Now, let us consider another process of dropping a different book B , instead of A , and see what happens. The subsequent evolution in this case is similar to the case with A . In fact, if the masses of A and B are the same, then the masses of the black holes after absorbing these books will be the same, so the final state radiations obtained after evaporation of these black holes are also expected to be the same, because the form of radiation

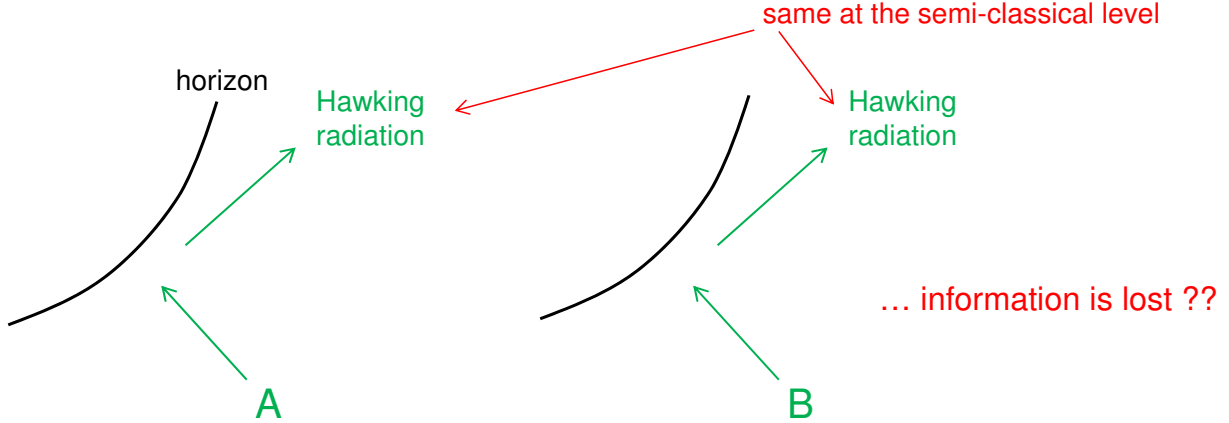


Figure 2: If we drop two different objects (e.g. A and B) into a black hole, the final states seem to be identical at the level of semi-classical approximation, leading to information loss. This is believed *not* to be the case at the full quantum level—final state Hawking radiation contains the full information about the initial state (A or B) in the form of subtle quantum correlations between radiation quanta.

depends only on the mass of a black hole in the semi-classical approximation [16], which was believed to be correct for large systems like black holes.

If the final state radiations are really identical—regardless of the details of the books—then this would imply that “information is lost.” Namely, one cannot *in principle* recover what was the initial state just by looking at the final state of the system; see Fig. 2. (In technical terminology, it is said that unitarity is violated.) Who cares? We care! In any other situation in physics, we never encounter this kind of phenomenon. For example, Newtonian mechanics is deterministic, meaning that if we have perfect knowledge about the current state of a system, then we can know its future and past by evolving the equation of motion forward and backward in time. Even in quantum mechanics, the Schrödinger equation is deterministic, so that perfect knowledge of a quantum state should allow us to infer its future and past (although, in practice, it is impossible to have such knowledge). To accept the information loss, we need to give up usual (unitary) quantum mechanics.

Following recent progress in understanding quantum gravity, especially the discovery of the anti de Sitter/conformal field theory duality [17] (which allow us to map certain gravitational systems into known, unitary theories), theorists now do not think such information loss will actually occur. We now think that the final state radiations obtained from evaporation of the black holes that have absorbed book A and B are, in fact, slightly different—different in quantum entanglement between many quanta in the radiation. It is simply that when the semi-classical approximation is adopted, which discards all the information on such quantum correlations, the two final states in Fig. 2 *look* the same. This situation is, in fact, not much different from burning a book in a (fictitious) classical, Newtonian world. Even in this case, the final states of burning book A and

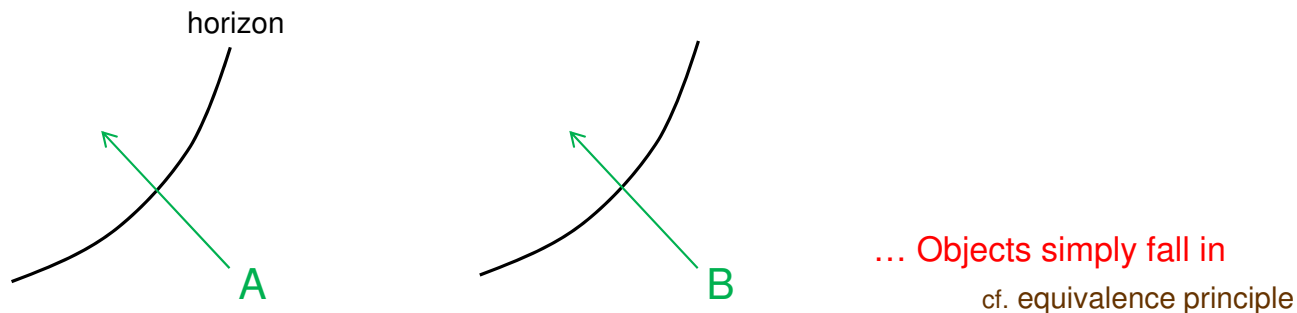


Figure 3: If we observe the same process as in Fig. 2 from a falling observer’s point of view, the falling object (A or B) simply passes the black hole horizon without any disruption. The full information about the object (in fact, the object itself) will therefore be inside the horizon at late times from this observer’s viewpoint.

B look quite the same—some ashes and dirty air—but if we know the states precisely enough, specifically the locations and velocities of all the molecules, then the information about the initial states should still be there: we must be able to solve the Newton equation backward in time to see if the initial book was A or B . In this sense, black holes are quite “conventional” objects—they simply “burn” information (or “scramble” it in technical terms).

A puzzling thing occurs, however, if we observe *the same phenomenon* from the viewpoint of an observer who is falling into the black hole with a book. In this case, the equivalence principle says that the book does not feel gravity (except for the tidal force which is tiny for a large black hole), so that it simply passes through the black hole horizon without any disruption; see Fig. 3. This implies that all the information about the book (in fact, the book itself) will be *inside* the horizon at late times. On the other hand, we have just argued that from a distant observer’s point of view, the information will be *outside*—first on (or more precisely, near) the horizon and then in Hawking radiation emitted from the black hole. Which is correct?

One might think that the information is simply duplicated: one inside and the other outside. This, however, cannot be the case. Quantum mechanics prohibits faithful copy of full quantum information, due to the no-cloning theorem [18]. (A simple way of seeing this is that if a quantum state could be duplicated, then one would be able to measure complementary quantities, e.g. position and momentum, in each copy, contradicting the uncertainty principle.) Therefore, it seems that the two pictures by the two observers cannot both be correct.

The solution to this puzzle is quite interesting—*both pictures are correct, but not at the same time*. The point is that one cannot be *both* a distant observer *and* a falling observer at the same time. If you are a distant observer, the system looks as in Fig. 2 so that the information will be outside, while if you are a falling observer, then the system appears as in Fig. 3 and the information (the book itself) will be inside. There is no inconsistency in either picture; only

if you artificially “patch” the two pictures, which you cannot physically do, then the apparent inconsistency of information duplication occurs. Note that for this argument, the existence of the horizon is crucial—because of it, the distant and falling observers cannot compare their findings about the location of information, avoiding contradiction in a single picture. This surprising aspect of a system with gravity is called black hole complementarity [13].

An important lesson from the above analysis of black holes is that quantum states must be defined carefully in a system with gravity. From a general relativistic point of view, there is nothing wrong with defining quantum states on late-time spacelike hypersurfaces—often called nice slices—on which the information exists *both* in Hawking radiation *and* internal space. This, however, leads to (fictitious) duplication of quantum information initially carried by a falling object: *including both Hawking radiation and the inside spacetime region within a single description is overcounting*. To avoid this problem, Hilbert space for the quantum states must be restricted to the one associated with appropriate spacetime regions: “your side” of the horizon. For example, if you include Hawking radiation as well as the horizon degrees of freedom in your description, i.e. if you are a distant observer, then the internal space of the black hole *literally does not exist*—including it would violate the laws of quantum mechanics.

3.2 The multiverse as a quantum mechanical universe

Let us now consider eternally inflating spacetime. Because of accelerating expansion, an inflationary space has a horizon—we cannot see an object further than a certain distance, called the de Sitter horizon, because the expansion of space makes any signals from such an object unreachable to us. This situation is simply the “inside out” version of the black hole case viewed from a distant observer! In the black hole case, we were staying outside the horizon, while now inside; but the basic thrust is the same—*spacetime on the other side of the horizon does not exist*. Specifically, in an eternally inflating spacetime, if you include Gibbons-Hawking radiation (an analogue of Hawking radiation in the black hole case), then you should not include the region outside the horizon in your description of quantum states. More precisely, the horizon here is the stretched apparent horizon, which, unlike the event horizon, can be defined locally without knowing what happens in the future (and which exists not only in an exponentially expanding de Sitter space but also in other cosmologically relevant spacetimes) [10, 11].¹

Our current universe is in a phase of accelerating expansion, so there is a de Sitter horizon at about $4.2 \text{ Gpc} \simeq 1.3 \times 10^{23} \text{ km}$ away from us. Its consistent quantum mechanical description, therefore, requires us *not* to include spacetime outside this horizon. Then what is the multiverse, which we thought exists further away beyond the horizon? The answer is: the probability! Given

¹Possible relevance of black hole complementarity in eternal inflation has been noted earlier in Ref. [19], although its explicit implementation is different from the one considered here.

simple initial conditions such as an eternally inflating state $|\Sigma\rangle$, the quantum state evolves into a superposition of various different cosmic configurations, as shown in Eq. (5). Each component (or term) of the state corresponds to a quantum state on a well-defined semi-classical geometry, *defined only on your side (which we will refer to as inside hereafter) of and on the apparent horizon*. In particular, these terms will contain universes like our own, but having different ρ_Λ ; and the coefficient of each such term $c(\rho_\Lambda)$ will give, after taking into account an appropriate weight for ourselves to emerge, the probability density of finding a particular value of ρ_Λ :

$$P(\rho_\Lambda) \propto |c(\rho_\Lambda)|^2. \quad (6)$$

(The issue of defining probabilities will be discussed in more detail in Section 3.4. For an explicit calculation of $P(\rho_\Lambda)$ in the present context, see Ref. [20].) To put it simply, *the multiverse lives in probability space*.

Formally, the construction of Hilbert space for the multiverse state $|\Psi(t)\rangle$ implementing the picture described above can be made quite analogously to the usual Fock space construction in quantum field theory [10, 11]. For a fixed semi-classical geometry \mathcal{M} (or more precisely, a set of fixed semi-classical geometries $\mathcal{M} = \{\mathcal{M}_i\}$ having the same horizon $\partial\mathcal{M}$), the Hilbert space is given by

$$\mathcal{H}_{\mathcal{M}} = \mathcal{H}_{\mathcal{M},\text{bulk}} \otimes \mathcal{H}_{\mathcal{M},\text{horizon}}, \quad (7)$$

where $\mathcal{H}_{\mathcal{M},\text{bulk}}$ and $\mathcal{H}_{\mathcal{M},\text{horizon}}$ represent Hilbert space factors associated with the degrees of freedom inside and on the apparent horizon $\partial\mathcal{M}$. The dimensions of these spaces are given by

$$\dim \mathcal{H}_{\mathcal{M},\text{bulk}} = \dim \mathcal{H}_{\mathcal{M},\text{horizon}} = \exp\left(\frac{\mathcal{A}_{\partial\mathcal{M}}}{4}\right), \quad (8)$$

where $\mathcal{A}_{\partial\mathcal{M}}$ is the area of the horizon in units of l_P . The fact that the maximum number of degrees of freedom (i.e. the logarithm of the dimension of the Hilbert space) scales with the area, rather than the volume, is a manifestation of the holographic principle [12], which roughly says that the number of degrees of freedom that can be put in a fixed region in a theory with general covariance is limited by the area of the surface bounding it. The full Hilbert space for dynamical spacetime is then given by the direct sum of the Hilbert spaces for different \mathcal{M} 's

$$\mathcal{H} = \bigoplus_{\mathcal{M}} \mathcal{H}_{\mathcal{M}}, \quad (9)$$

which is analogous to the Fock space construction in quantum field theory: $\mathcal{H}_{\text{QFT}} = \bigoplus_{n=0}^{\infty} \mathcal{H}_{1\text{P}}^{\otimes n}$, where $\mathcal{H}_{1\text{P}}^{\otimes n}$ is the n -particle Hilbert space. In addition, the complete Hilbert space for quantum gravity must contain “intrinsically quantum mechanical” states, associated with spacetime singularities:

$$\mathcal{H}_{\text{QG}} = \mathcal{H} \oplus \mathcal{H}_{\text{sing}}, \quad (10)$$

where $\mathcal{H}_{\text{sing}}$ represents the Hilbert space for the singularity states. The evolution of the multiverse state $|\Psi(t)\rangle$ is deterministic and unitary in \mathcal{H}_{QG} , but not in $\mathcal{H}_{\mathcal{M}}$ or \mathcal{H} .

3.3 “Reference frame dependence” of the concept of spacetime

In the construction of Hilbert space in the previous section, we invoked apparent horizons. In the cosmological context, however, the locations of these horizons are “observer dependent.” For example, the location of a de Sitter horizon depends on a spatial point we consider to be the center. What does this really mean?

What we are actually doing here is fixing *a reference frame*, including “the origin of the coordinates.” It is well known that to do Hamiltonian quantum mechanics—which we are doing here—we must fix all the “gauge redundancies,” the redundancies of describing the same system in different ways. A theory of gravity has huge redundancies associated with general coordinate transformations, and fixing a reference frame (more precisely, electing a local Lorentz frame) is precisely a way to eliminate these redundancies and to extract physical information, i.e. causal relations among events which are invariant under general coordinate transformations. This is so important that we repeat it again—*we need to fix a reference frame when we describe a system with gravity quantum mechanically*. This makes the apparent horizon well-defined: it is the horizon as viewed from the center (the “origin”) of the chosen reference frame. Once a reference frame is chosen, the location of a physical object with respect to its center has a physical meaning; in particular, spacetime outside the horizon does not exist, as we argued earlier.

What happens if we change the reference frame, e.g. by a spatial translation or boost? As in any symmetry transformation, this operation must be represented by a unitary transformation in the entire Hilbert space \mathcal{H}_{QG} . There is, however, no reason why it must be represented in each component $\mathcal{H}_{\mathcal{M}}$. In particular, the transformation can in general mix elements in different $\mathcal{H}_{\mathcal{M}}$ (as well as those in $\mathcal{H}_{\text{sing}}$). Moreover, even if the transformation maps all the elements in $\mathcal{H}_{\mathcal{M}}$ onto themselves for some \mathcal{M} , there is no reason that it should not mix the degrees of freedom associated with $\mathcal{H}_{\mathcal{M},\text{bulk}}$ and $\mathcal{H}_{\mathcal{M},\text{horizon}}$.

This has a dramatic consequence on the notion of spacetime in a theory with gravity [11]. Consider a state in space with accelerating expansion (de Sitter space). If we change the reference frame by performing a spatial translation, then (a part of) the degrees of freedom associated with *internal spacetime* in the original state must be described as the *horizon* degrees of freedom after the reference frame change, and vice versa; see Fig. 4 (left). This implies that what is spacetime and what is not (in this case, the horizon) depends on a reference frame! A more drastic situation occurs when there is a black hole. Consider a reference frame in which the center stays outside the horizon at late times. In this reference frame, the internal space of the black hole does not exist, as argued before. Now, let us change a reference frame by performing a boost at some early time so that the center of the new reference frame enters into the black hole at late times; Fig. 4 (right). In this case what was described as the horizon degrees of freedom (and Hawking radiation) in the old reference frame is now described as the internal spacetime of the black hole (and the

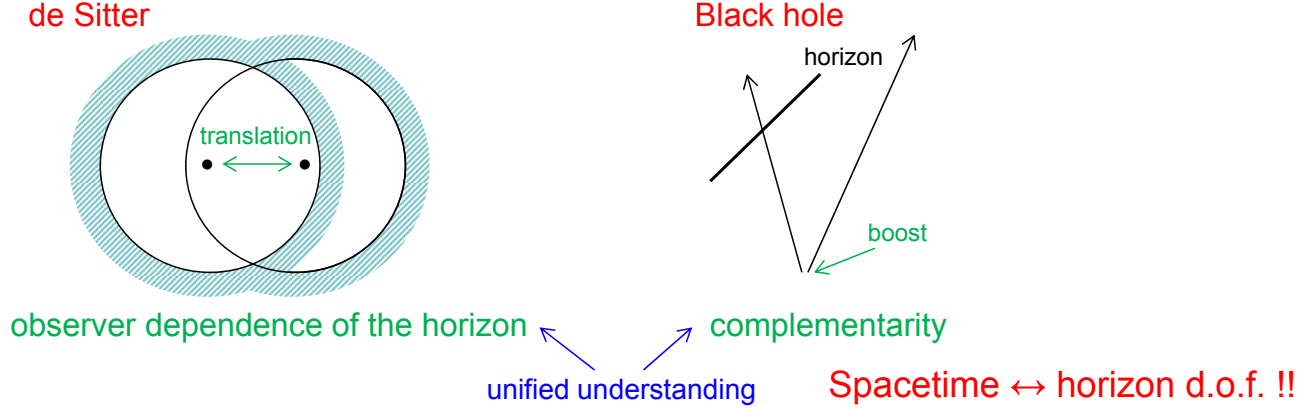


Figure 4: If we change a reference frame by a spatial translation in de Sitter space, what is described as spacetime in one reference frame is described (in part) as the horizon degrees of freedom in the other frame. In a system with a black hole, a large boost at an early time similarly transforms spacetime (and the singularity) inside the black hole into the horizon degrees of freedom (and Hawking radiation). These phenomena are nothing but the “observer dependence” of horizons and black hole complementarity, which can be understood as special cases of the more general transformation associated with reference frame changes.

singularity)! Note that in either of these examples, before and after the reference frame change, we are describing *the same (entire) physical system, not only their parts*. It is simply that what is spacetime in one reference frame is something else (a horizon, singularity, etc) in another—the *concept of spacetime depends on the reference frame*.

The phenomena we have just seen are exactly the “observer dependence” of horizons and black hole complementarity. They can, therefore, be understood in a unified manner as special cases of the general transformation (i.e. reference frame changes) considered here. They arise because changes of the reference frame are represented in Hilbert space \mathcal{H}_{QG} , which contains components $\mathcal{H}_{\mathcal{M}}$ that are defined only in restricted spacetime regions because of the existence of horizons.

The transformation described here can be viewed as an extension of the Lorentz/Poincaré transformation in the quantum gravitational context [11]; indeed, it is reduced to the standard Poincaré transformation of special relativity in the limit $G_N \rightarrow 0$, where G_N is the Newton constant. This is precisely analogous to the fact that the Lorentz transformation (which is a subgroup of the Poincaré transformation) is viewed as an extension of the Galilean transformation, which arises as the $c \rightarrow \infty$ limit of the Lorentz transformation, where c is the speed of light. In the Galilean transformation a change of the reference frame leads only to a constant shift of all the velocities, while in the Lorentz transformation it also alters temporal and spatial lengths (time dilation and Lorentz contraction) and makes the concept of simultaneity relative. With gravity, a change of the reference frame makes even the concept of spacetime relative—general relativity makes things really relative in the quantum context! See Fig. 5 for the summary of these relations.



Figure 5: The transformation described here is reduced to the Poincaré transformation of special relativity in the limit $G_N \rightarrow 0$, whose subgroup—the Lorentz transformation—is reduced to the Galilean transformation of Newtonian mechanics in the limit $c \rightarrow \infty$. Physical descriptions of nature become more relative as more fundamental constants of nature are turned on, as represented by the left-pointing arrows. With gravity ($G_N \neq 0$), a change of the reference frame makes even the concept of spacetime relative.

3.4 Unification of the eternally inflating multiverse and many worlds in quantum mechanics

Having defined the multiverse state $|\Psi(t)\rangle$, physical questions can now be answered following the rule of quantum mechanics. An important point here is that the “time” t in quantum gravity is simply an auxiliary parameter introduced to describe the “evolution” of the state, exactly like a variable t used in a parametric representation of a curve on a plane, $(x(t), y(t))$. The physical information is only in *correlations* between events, like correlations between x and y in the case of a curve on a plane [21]. Specifically, time evolution of a physical quantity X is nothing more than a correlation between X and a quantity that can play the role of time, such as the location of the hands of a clock or the average temperature of CMB in our universe.

Any physical question can then be phrased as: given condition A we specify, what is the probability for an event B to occur? For example, one can specify a certain “premeasurement” situation A_{pre} (e.g. the configuration of an experimental apparatus and the state of an experimenter before measurement) as well as a “postmeasurement” situation A_{post} (e.g. those after the measurement but without specifying outcome) as $A = \{A_{\text{pre}}, A_{\text{post}}\}$, and then ask the probability of a particular result B (specified, e.g., by a physical configuration of the pointer of the apparatus in A_{post}) to be obtained. In the context of the multiverse, the relevant probability $P(B|A)$ is given by [10, 11]:

$$P(B|A) = \frac{\iint dt_1 dt_2 \langle \Psi(0) | U(0, t_1) \mathcal{O}_{A_{\text{pre}}} U(t_1, t_2) \mathcal{O}_{A_{\text{post}} \cap B} U(t_2, t_1) \mathcal{O}_{A_{\text{pre}}} U(t_1, 0) | \Psi(0) \rangle}{\iint dt_1 dt_2 \langle \Psi(0) | U(0, t_1) \mathcal{O}_{A_{\text{pre}}} U(t_1, t_2) \mathcal{O}_{A_{\text{post}}} U(t_2, t_1) \mathcal{O}_{A_{\text{pre}}} U(t_1, 0) | \Psi(0) \rangle}. \quad (11)$$

Here, $U(t_1, t_2) = e^{-iH(t_1-t_2)}$ is the “time evolution” operator with H being the Hamiltonian for the entire system, and \mathcal{O}_X is the operator projecting onto states consistent with condition X . Note that since we have already fixed a reference frame, conditions A_{pre} and A_{post} in general must involve specifications of the locations and velocities of physical objects *with respect to the origin of the coordinates*, in addition to those of physical times made through configurations of non-static objects (e.g. the hands of a clock or the status of an experimenter).

The integrations over the “time” variable t in Eq. (11) must be taken from $t = 0$, where the initial condition for $|\Psi(t)\rangle$ is specified,² to $t = \infty$, because conditions A_{pre} and A_{post} may be satisfied at any values of $t > 0$ (denoted by t_1 and t_2 in the equation). Despite the integrals running to ∞ , the formula of Eq. (11) does not involve infinities, which would arise if an event occurred infinitely many times with a finite probability. This is because given a generic initial condition, the multiverse state $|\Psi(t)\rangle$ at late times will evolve into a superposition of terms corresponding to supersymmetric Minkowski space (certain highly symmetric space with $\rho_\Lambda = 0$) or spacetime singularity:

$$|\Psi(t)\rangle \xrightarrow{t \rightarrow \infty} \sum_i a_i(t) |\text{supersymmetric Minkowski space } i\rangle + \sum_j b_j(t) |\text{singularity state } j\rangle, \quad (12)$$

since these are the only absolutely stable states in the string landscape; the coefficients of the other components, including the ones which are selected by $\mathcal{O}_{A_{\text{pre}}}$ and $\mathcal{O}_{A_{\text{post}}}$, decay exponentially. This makes the meaning of eternal inflation clear. It is the picture obtained by focusing on a component staying in a meta-stable de Sitter state, whose coefficient, however, is decaying exponentially with t . In particular, expansion of space does not imply the increase of probability.

Equation (11) is our final formula for the probabilities. This is essentially the Born rule; indeed, one can show that the formula is reduced to the standard Born rule under the usual situation of a terrestrial experiment. There is no freedom of choosing one’s own (arbitrary) definition of probabilities, and there is no ambiguity associated with spatial points separated beyond the horizon, as spacetime beyond the horizon simply does not exist. Furthermore, the formula gives a well-defined, finite answer to any physical question we ask. Therefore, ...

The measure problem in eternal inflation is solved.

We emphasize that the uniqueness of the framework (for a given Hilbert space, which we take as in Eq. (10)) rests crucially on the specification of a reference frame, including its origin/center p . In particular, this requires us to specify ranges of location and velocity in which physical objects must lie *with respect to* p , in specifying A and B . This eliminates the ambiguity associated with how these objects must be counted. Of course, there is still a freedom of where we put these objects; for example, we could put them at p or some other point at rest, or could specify a phase space region in which they must be. But this is the freedom of questions one may ask, and not that of the framework itself. (And the final answer does not depend on the location/velocity of reference

²This is the initial condition for a state whose future evolution we want to follow (analogous to the initial condition of a dynamical system that we want to solve in Newtonian mechanics), and *not* (necessarily) the initial condition for the entire multiverse; namely, $|\Psi(0)\rangle$ here can simply be a component of the entire multiverse state at some particular moment. The real “beginning” of the quantum universe, i.e. the *ultimate* boundary condition for the *entire* multiverse state, is still an important open issue. (Note added: for a recent proposal on this issue within the framework discussed here, see Ref. [24].)

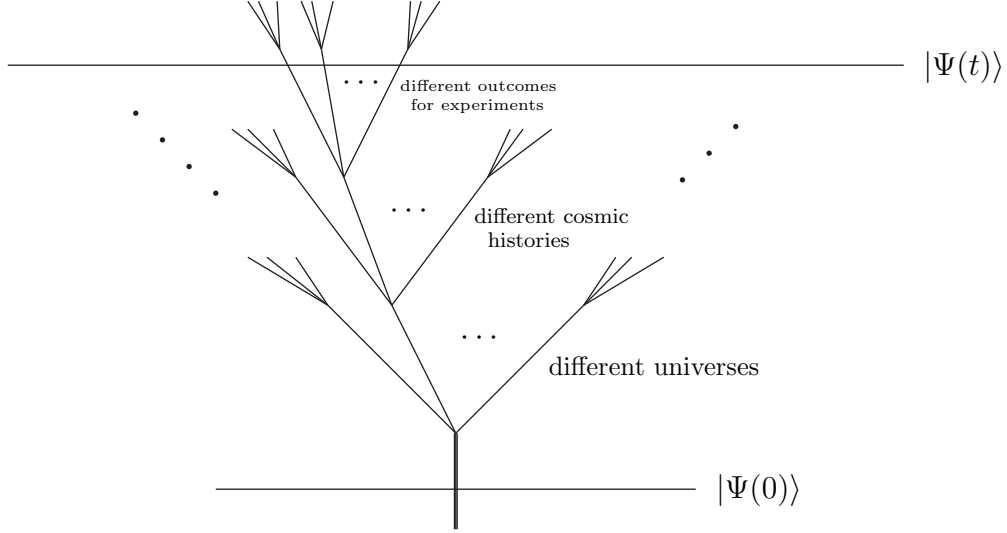


Figure 6: A schematic picture for the evolution of the multiverse state $|\Psi(t)\rangle$. As t increases, $|\Psi(t)\rangle$ evolves into a superposition of states in which various bubble universes nucleate in various spacetime locations. Each of these states then evolves further into a superposition of states representing various possible cosmic histories, including different outcomes of experiments performed within that universe.

point p , i.e. the overall relative location/velocity between p and the specified configurations in A and B , if the multiverse state $|\Psi(t)\rangle$ is invariant under the corresponding reference frame changes.)

The framework presented here provides a complete account for quantum measurement in the multiverse. Suppose the initial state $|\Psi(0)\rangle$ is in an eternally inflating phase. This state then evolves into a superposition of states in which various bubble universes nucleate in various spacetime locations. As t increases, a component representing each universe further evolves into a superposition of states representing various possible cosmic histories, including different outcomes of “experiments” performed within that universe. (These “experiments” may, but need not, be scientific experiments—they can be any physical processes.) For large t , the multiverse state $|\Psi(t)\rangle$ will therefore contain an enormous number of terms, each of which represents a possible world that may arise from $|\Psi(0)\rangle$ consistently with the laws of physics. While evolving, the multiverse state experiences both “branching” [22] and “amplification” [23], responsible, respectively, for the realization of various possible outcomes and the appearance of a classical world (selecting measurement bases) in each of them. Note that since the Hilbert space dimensions of $\mathcal{H}_{\text{Minkowski}}$ and $\mathcal{H}_{\text{sing}}$ —into which the multiverse state is evolving—are infinite, these different worlds do not recombine; they really branch into different worlds. A schematic picture for the evolution of the multiverse state is depicted in Fig. 6.

Our probability formula, Eq. (11), can be used to answer questions both regarding global properties of the universe and outcomes of particular experiments. For example, given premeasurement

situation A_{pre} , one can ask the probability of finding the vacuum energy ρ_Λ in a certain range or obtaining a particular outcome for an experiment performed in the laboratory, just by adopting different conditions A_{post} and B . This, therefore, provides complete unification of the eternally inflating multiverse and many worlds in quantum mechanics:

$$\boxed{\text{Multiverse} = \text{Quantum many worlds.}}$$

These two are really the same thing—if one asks a question about a global property of the universe, then it is called the multiverse, while if one asks a question about outcomes of an experiment/event in our everyday life, then it is called quantum many worlds. They simply refer to the same phenomenon occurring at (vastly) different scales.

4 Conclusions

In the past decade or two, a revolutionary change of our view of nature has started taking a concrete form—our universe may be one of the many in the vast multiverse. This view is motivated both observationally and theoretically: the discovery of the nonvanishing vacuum energy density in our universe and the string landscape/eternal inflation picture. In this article, we have seen that this also comes with a dramatic new view of spacetime and gravity, which was forced to resolve the puzzle of predictivity crisis that existed in the conventional, semi-classical view of eternally inflating spacetime. We have presented a remarkably simple framework that is applicable to physics at all scales: from the smallest (Planck length) to the largest (multiverse). We have seen that two seemingly different concepts—the multiverse and quantum many worlds—are, in fact, the same. They simply refer to the same phenomenon occurring at different length scales.

It is, indeed, quite striking that quantum mechanics does not need any modification to be applied to phenomena at such vastly different scales. In the 20th century, we have witnessed the tremendous success of quantum mechanics, following its birth at the beginning. In the early 21st century, quantum mechanics still seems to be giving us an opportunity to explore deep facts about nature, such as spacetime and gravity. Does quantum mechanics break down at some point? We don't know. But perhaps, exploring the ultimate beginning of the multiverse might provide a key to answer that question.

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